T_y. (Note that his $\sin 2\theta \cos\phi$ is better restored to direction cosine form before comparing.) Similarly, it is found that L_2 = his T_x, and L_3 = his $-T_z$, as should be the case because of different axis orientation conventions.

In summary, Eq. (7) is a convenient matrix form for the gravitational torque in a completely general field. It reduces to Schlegel's under the conditions he assumes, but is somewhat more general in that it does not imply that body axes are principal axes, and it allows the body orientation to be measured from a frame with one more degree of rotational freedom. Furthermore, the body orientation is not tied to any specific parametrization of the direction cosine matrix.

I fully agree that the magnitude of the torque correction from oblateness is very small, but I cannot concur with Schlegel that it yet has been established that it "can be safely ignored save in high precision studies." Although I suspect he is correct, it should be recognized that the oblateness effect will appear not only as an "external torque" in the dynamical equations, but also as a parametrix excitation. Under these conditions it is conceivable that it results in changes in stability characteristic or response amplitude to a degree quite unanticipated from its numerical magnitude. I believe that further studies are required to determine whether this kind of behavior can arise in situations which are of practical interest.

References

[1] Schlegel, L. B., "Contribution of Earth Oblateness to Gravity Torque on a Satellite," *AIAA Journal*, Vol. 4, No. 11, Nov. 1966, pp. 2075–2077.

[2] Pengelley, C. D., "Gravitational Torque on a Small Rigid Body in an Arbitrary Field," *ARS Journal*, Vol. 32, 1962, pp. 420–422.

Comment on "Large Deflection of Rectangular Sandwich Plates"

C. V. Smith*
Georgia Institute of Technology, Atlanta, Ga.

THIS Comment concerns the statement of the boundary conditions presented by the authors[1] in their Eq. (3) and rewritten below in a different form:

$$\text{at } \xi = \pm 1 \quad U = V = 0 \qquad W = \alpha = \beta = \partial W/\partial \xi = 0$$

$$\text{at } \eta = \pm 1 \quad U = V = 0 \qquad W = \alpha = \beta = \partial W/\partial \eta = 0$$

The equations involving U and V represent possible statements of the necessary two boundary conditions for the inplane system. The equations involving W, α, β, and $\partial W/\partial \xi$ ($\partial W/\partial \eta$) would then represent four boundary conditions for the bending. However, plate bending is described by a sixth-order set of equations, so that there can be just three boundary conditions on each edge. The extra and incorrect boundary conditions are the following:

$$\text{at } \xi = \pm 1 \qquad \partial W/\partial \xi = 0$$

$$\text{at } \eta = \pm 1 \qquad \partial W/\partial \eta = 0$$

The stress-strain-displacement relations for transverse shear are

$$S_x/G_c h = \gamma_{xz} = \alpha + \theta\, \partial W/\partial \xi$$

$$S_y/Q_c h = \gamma_{yz} = \beta + \theta\, \partial W/\partial \eta$$

Received September 25, 1967.

* Assistant Professor of Aerospace Engineering. Member AIAA.

where S_x and S_y are the transverse shear forces, and γ_{xz} and γ_{yz} are the transverse shear strains. These equations show that requiring both α and $\partial W/\partial \xi$ to vanish forces zero transverse shear strain at the boundaries $\xi = \pm 1$. Since the edge transverse shear strains are not zero for a sandwich plate, the center deflection will actually be larger than reported by the authors.

Reference

[1] Kan, H. and Huang, J., "Large Deflection of Rectangular Sandwich Plates," *AIAA Journal*, Vol. 5, No. 9, Sept. 1967, pp. 1706–1708.

Reply by Author to C. V. Smith

Ju-chin Huang*
Tennessee Technological University, Cookeville, Tenn.

SINCE we can always combine α with $\partial W/\partial \xi$ and β with $\partial W/\partial \eta$ to obtain transverse shear strains, boundary conditions (3) of our Note[3] are not independent. They are correct conditions, since the solutions of our Note are given for a rectangular sandwich plate with rigidly clamped edges. The conditions characterizing a rigidly clamped edge parallel to the coordinate-axes are zero deflection and zero slope of the middle surface along the edge and zero rotation of the cross section making up the boundary. These requirements certainly force the transverse shear strain to zero at the boundaries. It can be easily seen by considering the deflection of a cantilever sandwich beam due to deformation associated with the shear stress as shown in Fig. 1. Element A, on the neutral axis, which is originally rectangular, changes into a rhombus, but element B near the clamped end remains unchanged. Hence we can simply assume that the transverse shear strain equals to zero at the restrained end.

In the early works of Hoff,[1,2] similar boundary conditions have been used for the bending of a cantilever sandwich beam and bending of rectangular sandwich plate with edges clamped. The transverse shear strains vanish at the restrained boundaries for both cases.

For the sake of simplicity, we can always relax the boundary conditions to some extent by letting the edge slopes of the middle surface of the plate be different from zero. Naturally this assumption will lead to slightly larger center deflection.

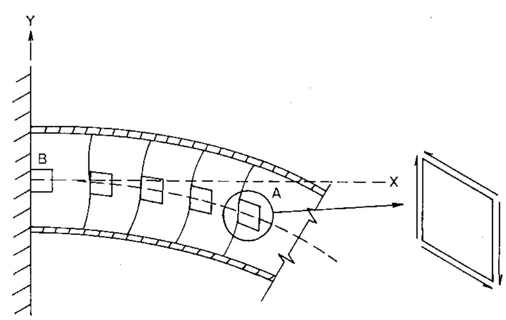

Fig. 1 Deflection of cantilever sandwich beam.

References

[1] Hoff, N. J., *The Analysis of Structure*, Wiley, New York, 1956, pp. 180–193; also Hoff, N. J. and Mantner, S. E., "Bending and Buckling of Sandwich Beams," *The Journal of the Aeronautical Sciences*, Vol. 15, No. 12, 1948, p. 707.

Received October 26, 1967.

* Assistant Professor, Dept. of Engineering Science.

[2] Hoff, N. J., "Bending and Buckling of Rectangular Sandwich Plates," TN 2225, 1950, NACA.

[3] Kan, Han-pin and Huang, Ju-chin, "Large Deflection of Rectangular Sandwich Plates," *AIAA Journal*, Vol. 5, No. 9, Sept. 1967, pp. 1706–1708.

Comments on "Natural Vibrations of a Stiffened Pressurized Cylinder with an Attached Mass"

Robert Dan Smith*
Lockheed Palo Alto Research Facility, Palo Alto, Calif.

THE following comments concern the article by I. U. Ojalvo and M. Newman[1] which appeared in the June 1967 issue of the *AIAA Journal*. In this article the authors investigate theoretically and experimentally the free vibrations of an internally pressurized, cylindrical shell to which is attached a concentrated mass. In a number of cases the shell is stiffened by the addition of rings that subdivide the shell into bays. In the remaining cases the shell is further stiffened by the addition of stringers.

First, the applicability of the assumption of zero axial restraint, for the edges of the bays investigated in the analysis, is open to question. Each of the intermediate bays of the shell is continuously connected to the two adjacent bays. This situation is not altered by the attachment, at discrete rivet locations, of rings that have high out-of-plane twisting compliances. If an intermediate bay is excited in a frequency range where the radial motion of the shell is dominant in comparison to the axial motion, as in the experiments, then $|u|]_{\text{adjacent bays}} \ll |u|]_{\text{excited bay}}$ since $|w|]_{\text{adjacent bays}} \ll |w|]_{\text{excited bay}}$. This indicates the presence of appreciable axial forces at the edges of the excited bay, which is investigated in the analysis.

In addition, upon selecting an analytical model of the coupled shell-attached mass system involving a shell having axially unrestrained edges, the authors do not state whether a rigid body mode[2] of the form $(u, v, w) = (1, 0, 0)$ has or has not been included in the modal expansion given for the shell displacements. Since, in the analysis, the shell is axially unrestrained, such a mode is necessary in order to allow a nonzero acceleration of the mass center of the shell, due to the axial force exerted by the attached mass. It should be noted, however, that this rigid body mode would not be present in the axial motion of the shell for the forced response problem, associated with the experiments, in which the analytical model has both the attached mass and the radial exciting force located at $x = L/2$.

Second, consider, as the authors do, only the case for which radial shell motion is symmetric about $\theta = 0$. Now there exists a one-to-one correspondence between the frequencies for the unweighted modes and those for the coupled system modes, i.e., the addition of the attached mass does not create any new frequencies but causes a downward perturbation of the unweighted system frequencies. In addition, for the case in which the forces exerted by the attached mass do not act through displacements of zero amplitude, no coupled system frequency may be equal to an unweighted system frequency. If this were not true, the attached mass, oscillating in a coupled system mode, could drive at resonance one of the unweighted shell modes. A comparison of Tables 2 and 3, for cylinder 1, reveals that there exist two coupled system frequencies below the lowest unweighted system frequency in apparent contradiction to the preceding statements. A possible interpretation of this result is that one of the set of coupled system modes, involving antisymmetric radial motion of the shell about $\theta = 0$, has been excited. Such modes have not been considered by the authors. A typical mode of this group involves motion of the attached mass in the circumferential direction as well as shell displacements which can be described by an expansion in the unweighted antisymmetric modal displacements[3] about $\theta = 0$, i.e.,

$$u = q_{kl}^{(1)} \sin(k\theta) \cos(\lambda_l x) \qquad v = -q_{kl}^{(2)} \cos(k\theta) \sin(\lambda_l x)$$

$$w = q_{kl}^{(3)} \sin(k\theta) \sin(\lambda_l x)$$

where $k = 0, 1, 2, 3, \ldots$ and $l = 1, 2, 3, \ldots$ are possible choices for k and l.

Third, for the forced response problem associated with the experiments, i.e., the attached mass and the radial exciting force located at $\bar{x} = L/2$, the axial displacement of the attached mass is equal to zero. However, for the general case of free vibration of the coupled system and $\bar{x} \neq L/2$, it is to be noted that as $\bar{n}$, the number of unweighted modes used in the analysis of the coupled system, becomes infinite, i.e., as one hopefully proceeds toward an exact solution, the axial displacement of the point mass

$$[u]_{\substack{\theta=0\\x=\bar{x}}}$$

given in Eq. (29) of Ref. 1 becomes infinite. Thus the kinetic energy of the system, as given in Eq. (28) of Ref. 1, also becomes infinite. Such difficulties tend to cast doubt on the validity of the analysis. The divergence of the modal representation for

$$[u]_{\substack{\theta=0\\x=\bar{x}}}$$

may be seen from the following considerations. Let $\tilde{u}$ be the axial displacement of the unweighted shell at $x = \bar{x}$, $\theta = 0$ due to an axial unit harmonic point force $e^{i\omega_c t}$. In addition, let the index j denote the free vibration modes of the unweighted shell. Utilizing the notation of Ref. 1, $\tilde{u}$ can be described in terms of these modes as

$$\tilde{u} = -\frac{1}{M_0\omega_c^2} + \sum_{j=1}^{\infty} \frac{[Q_j^{(1)} \cos(\lambda_j \bar{x})]^2}{M_j(\omega_j^2 - \omega_c^2)}$$

where M_0 is the mass of the shell and M_j is the generalized mass for mode j. The admittance quantity $\tilde{u}$ may be related to

$$[u]_{\substack{\theta=0\\x=\bar{x}}}$$

via the equation

$$[u]_{\substack{\theta=0\\x=\bar{x}}} = \tilde{u} \text{ (axial force on the cylinder due to the point mass)}$$

Let us also consider the static problem of the same cylinder subjected to an axial point force, of unit magnitude, located $x = \bar{x}$, $\theta = 0$ and counterbalanced by a uniform body force distribution. Utilizing the modes of the unweighted shell, one can express the axial displacement $\hat{u}$ at $x = \bar{x}$, $\theta = 0$ as

$$\hat{u} = \sum_{j=1}^{\infty} \frac{[Q_j^{(1)} \cos(\lambda_j \bar{x})]^2}{M_j \omega_j^2}$$

One can also represent $\hat{u}$ in terms of a double Fourier series, as is done for similar problems in Ref. 4. The contribution to this series from the terms with sufficiently large values of the axial and circumferential Fourier indices (m, n) can be ex-

Received July 14, 1967.

* Research Scientist, Aerospace Sciences Laboratory.